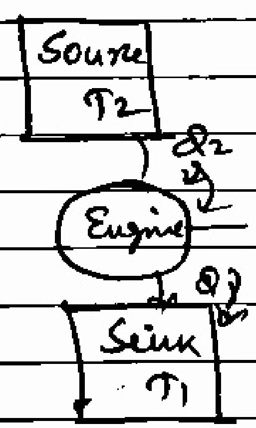
Carnot Cycle:

S Carnot who was basically an engineer he tried to perform a maximum efficiency engine (hypothetical), by using Source & Sink System. He took a Cylinder which wall was completely non-conducting and the base was fully conducting or diathermal fitted with a piston which is frictionless and containing 1 mole of ideal gas in it, this Cylinder or System is called Carnot heat engine. This system works ~~in~~ ~~betw~~ reversibly in cycles between two large heat reservoirs i.e Source (which works at high temperature) & another is Sink (Which works at low temperature).



In the above diagram Source is kept at higher temperature 'T_2' and Sink is at lower temperature 'T_1' when Q_2 heat is supplied to engine from Source it converts this into work and the rest heat i.e Q_1 is supplied to Sink.

Teacher's Signature : ______________

the Carbonot Cycle envolve four Steps i.e i) Isothermal expansion, ii) Isothermal Contraction or Compression iii) Adiabatic expansion & iv) Adiabatic Contraction a Compression, which we will descuss one by one separately.

Step I (Isothermal expansion) :→ The Cylinder Contain 1mole of ideal gas and placed with source at temperature T_2, it absorb the certain amount of heat i.e Q_2 and the the gas is allow to expand isothermally and reversibly at temperature T_2 of the source. The ~~system~~ expansion is Isothermal so the The temperature is remain constant and pressure volume changes from P_1V_1 to P_2V_2. In this situation change in internal energy (ΔE) becomes zero due to $\Delta T = 0$. Hence the work done after the absorption of heat is,

$$Q_2 = -\omega_1 = RT_2 \ln \frac{V_2}{V_1} \quad \text{—— I}$$

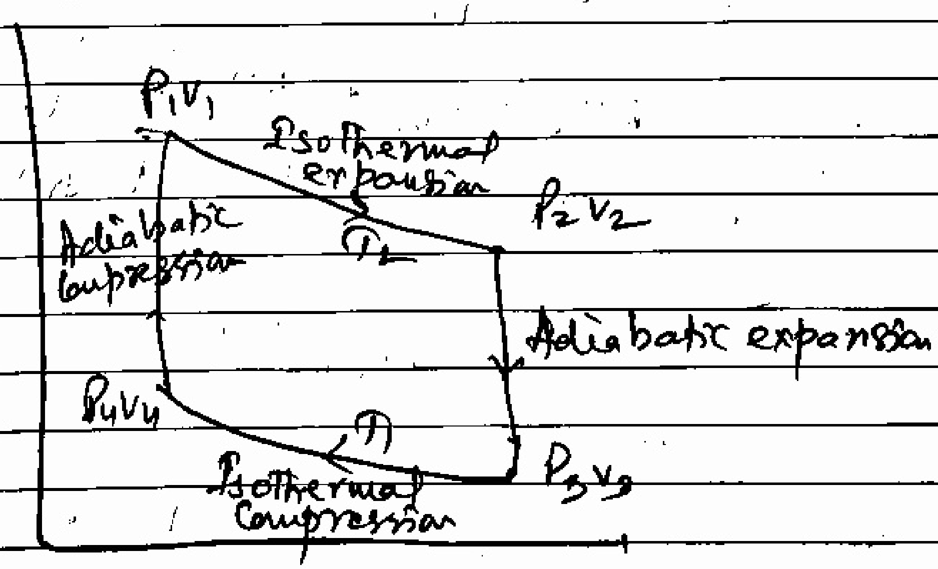
Step II Now the Cylinder is removed from the source and kept at such stand where the base is insulating means exchange of heat is not possible

Now the gas allow to expand ~~is~~ adiabatically & reversibly from volume V_2 to V_3 till that situation when temperature drop ~~from~~ to T_1 (the temperature of the sink). Here, the system is adiabatic $Q = 0$.

So work done by 1 mole of ideal gas: $-w_2 = +C_v(T_1 - T_2)$

$$-w_2 = +C_v(T_1 - T_2) \quad \text{---} \quad II$$

III Step: (Isothermal Compression): — Now the cylinder is removed from the stand and it is kept and sink (T_1). Now the gas is compressed reversibly & isothermally in such a way by which 'T' must be constant & pressure & volume changes from P_3V_3 to P_4V_4.



Now work done by 1 mole of ideal gas in isothermal compression, $w_3 = -Q_1$

$$w_3 = RT_1 \ln \frac{V_4}{V_3} \quad \text{---} \quad III$$

Step IV : (Adiabatic Compression) — Now the Cylinder is removed from the Sink and kept at Stand where the bottom is insulating. Now the gas is allowed to compress adiabatically & reversibly so that the temperature rises to till T_2 & the volume decreases from V_4 to V_1.

work done, $W_4 = C_v (T_2 - T_1)$. ——— IV

Now the System has returned to initial State The Carnot Cycle is Completed. So the work done by the Complete of Carnot Cycle can be written as

$$W = (-W_1) + (-W_2) + W_3 + W_4$$

$$= RT_2 \ln \frac{V_2}{V_1} - C_v (\cancel{T_2} - T_1) + RT_1 \ln \frac{V_4}{V_3} + C_v (\cancel{T_2} - T_1)$$

$$= RT_2 \ln \frac{V_2}{V_1} + RT_1 \ln \frac{V_4}{V_3} \quad \text{——— V}$$

Here V_2 & V_3 are on II Step & V_4 & V_1 are in IV Step since II & IV are adiabatic, therefore

$$T_2 V_2^{\gamma-1} = T_1 V_3^{\gamma-1} \quad \text{——— VI}$$

$$T_2 V_1^{\gamma-1} = T_1 V_4^{\gamma-1} \quad \text{——— VII}$$

When we devide eqn VI by VII we have

$$\frac{V_2}{V_1} = \frac{V_3}{V_4} \quad \text{——— VIII}$$

From equation V & VIII, Net work is:

$$W = RT_2 \ln \frac{V_2}{V_1} + RT_1 \ln \frac{V_1}{V_2} \quad \text{——— IX}$$

In equation I $Q_2 = RT_2 \ln \frac{V_2}{V_1}$

the efficiency of Carnot engine $\eta = \frac{W}{Q_2}$

$$\eta = \frac{W}{Q_2} = \frac{R(T_2 - T_1)\ln V_2/V_1}{RT_2 \ln V_2/V_1}$$

$$\eta = \frac{T_2 - T_1}{T_2} = 1 - \frac{T_1}{T_2}$$

$$\boxed{\eta = 1 - \frac{T_1}{T_2}}$$

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